

# The Possibility Of The Strict Global Thermodynamic Equilibrium In The Expanding Universe At Presence Of The Fundamental Scalar Field

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## Abstract

In the article it is shown that at presence of fundamental scalar fields determining the masses of the scalar charged particles the global thermodynamic equilibrium (GTE) is compatible with a process of the cosmological expansion of the statistical system.

*Keywords:* Early Universe, global thermodynamic equilibrium, relativistic kinetics, scalar interaction, cosmological acceleration.

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## 1. The General Relativistic Theory And The Thermodynamic Equilibrium

The basics of the general relativistic kinetic theory (GRKT) were stated in 60's in the works of N.A. Chernikov [1], [3] - [6], E.Tauber and J.Weinberg [2], A.A.Vlasov [7], R.Lindqvist [8] and others. Within the framework of the GRKT there were formulated the general relativistic kinetic equations on which basis the macroscopic transport equations were built as well as the theory of the global and local thermodynamic equilibrium of the statistical system in gravitational and electromagnetic fields was developed<sup>1</sup>. In the most complete and strict form the results of these researches are explicated in the following Author's works [9, 10, 11, 12, 13]. In this chapter we recap the results of these researches in the reduced form wherein we use more modern, canonical and explicitly invariant formulation of the kinetic theory, which was developed in Authors's works [14].

### 1.1. The Canonical Equations Of Motion In The Gravitational And Electromagnetic Fields And The Macroscopic Fluxes

The canonical equations of particle motion in canonically conjugate variables - coordinates  $x^i$  and

generalized momentum  $P_i$  have form:

$$p^i \equiv \frac{dx^i}{ds} = \frac{\partial H_a}{\partial P_i}; \quad \frac{dP_i}{ds} = -\frac{\partial H_a}{\partial x^i}, \quad (1)$$

where  $H_a(x, P)$  is a relativistically invariant Hamilton function,  $p^i$  is a kinematic momentum. The total derivative of an arbitrary function of the dynamic variables  $\Psi_a(x, P)$  with respect to the canonical parameter  $s$  is determined by the *Poisson brackets*:

$$\frac{d\Psi_a}{ds} = [H_a, \Psi_a] \equiv \left[ \frac{\partial H_a}{\partial P_i} \frac{\partial \Psi_a}{\partial x^i} - \frac{\partial H_a}{\partial x^i} \frac{\partial \Psi_a}{\partial P_i} \right]. \quad (2)$$

As a result of the equations (2) the Hamilton function itself is an integral of motion:

$$H_a(x, P) = \text{Const} = \frac{1}{2} m_a^2, \Rightarrow (p, p) = m_a^2, \quad (3)$$

where  $m_a$  is a rest mass of particles. The relation (3) is called the momentum normalization ratio or the equation of the mass surface<sup>2</sup>.

For the charged particles with masses  $m_a$  and electrical charges  $e_a$  in a gravitational fields having metrics  $g_{ik}(x)$  and electromagnetical fields with a vector potential  $A_i(x)$  the Hamilton function can

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<sup>1</sup>By the electromagnetic fields here one can imply also any vector fields.

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<sup>2</sup>Hereinafter  $(a, b) = g_{ik}a^ib^k$  is a scalar product of the momentums  $a$  and  $b$  relatively to the metrics  $g_{ik}$ .

be specified in the form<sup>3</sup>:

$$H_a(x, P) = \frac{1}{2}g^{ik}(P_i - e_a A_i)(P_k - e_a A_k) \\ \equiv \frac{1}{2}(P - e_a A, P - e_a A) \equiv \frac{1}{2}(P - e_a A)^2. \quad (4)$$

Let the statistical system in the gravitational field with metrics  $g_{ik}$  and in the electromagnetic field with the potential  $A_i$  consists of  $N$  sorts of the identical particles and  $f_a(x, p)$  are invariant functions of distribution of these particles in the phase space  $X \times P$ , ( $x^i$  – are coordinates,  $p^i$  is a momentum, so that:

$$n_a^i(x) = \int_{P(x)} f_a(x, p) p^i dP_a \quad (5)$$

there are  $a$ -sort particles' number flux densities

$$N_a(\tau) = \int_V n_a^i dV_i \equiv \int_V dV_i \int_{P(x)} f_a(x, p) p^i dP_a \quad (6)$$

and there is a number of  $a$ -sort particles on the spacelike hypersurface  $V : dx^i u_i = 0$ , where

$$dP_a = \frac{2S+1}{8\pi^3\sqrt{-g}} dp^1 dp^2 dp^3 dp^4 \delta(H_a) \\ \Rightarrow dP_a = \frac{2S+1}{4\pi^3\sqrt{-g}} \frac{dp^1 dp^2 dp^3}{p_4}, \quad (7)$$

$H_a(x, P)$  is a Hamilton function of  $a$ -sort particle in the gravitational  $g_{ik}$  and in the electromagnetic  $A_i$  fields,  $S$  – is a spin,  $p_4$  is a positive root of the mass surface equation (the covariant time component of the momentum):

$$H_a(x, P) = \frac{1}{2}m_a^2 \Rightarrow (p, p) = m_a^2. \quad (8)$$

### 1.2. The Common-relativistic Equations For Charged Particles

Let further the following reactions run in the statistical system:

$$\sum_{A=1}^m \nu_A a_A \rightleftharpoons \sum_{B=1}^{m'} \nu'_B a'_B \Rightarrow \sum_k \sum_{A=1}^N \nu_A^k a_A = 0, \quad (9)$$

where  $\nu_A$  is  $a_A$ -sort particle number of particles participating in the reaction;  $A$  is a number of the particle,  $||\nu_A^k||$  is an integer matrix of rank less than  $N$

<sup>3</sup>It is used everywhere the universal system of units  $G = c = \hbar = k = 1$  where  $k$  is a Boltzmann constant.

and the summation is carried out by all channels of reactions in which  $a$ -sort particles participate.

The distribution functions  $f_a(x, p)$  are determined by means of the relativistic kinetic equations:

$$[H_a, f_a] = I_a(x, P_a), \quad (10)$$

where  $J_a(x, P_a)$  are the collision integrals:

$$I_a(x, P_a) = - \sum_a \nu_a \int_a' \delta^4(P_F - P_I) \\ \times W_{IF}(Z_{IF} - Z_{FI}) \prod_{I,F}' dP; \quad (11)$$

where

$$W_{FI} = (2\pi)^4 2^{-\sum \nu_A + \sum \nu'_B} |M_{IF}|^2$$

is a scattering matrix for the channel of reactions (9), ( $|M_{IF}|$  are invariant amplitudes of scattering);

$$Z_{IF} = \prod_I f(P_A^\alpha) \prod_F [1 \pm f(P_B^{\alpha'})];$$

$$Z_{FI} = \prod_I [1 \pm f(P_A^\alpha)] \prod_F f(P_B^{\alpha'}),$$

sign “+” corresponds to bosons, “-” corresponds to fermions (details see in [11, 12]).

### 1.3. The Entropy Of The Statistical System

The entropy of the statistical system on the hypersurface  $V(\tau)$  is equal to:

$$S(\tau) = \sum_a S_a(\tau) = \sum_a \int_{V(\tau)} u^i dV_i \times \\ \int_{P_a(X)} [\pm(1 \pm f_a) \ln(1 \pm f_a) - f_a \ln f_a] dP_a, \quad (12)$$

where  $\tau = \int u_i dx^i$  is a proper time of an observer. The total derivative of the system entropy with respect to the time  $\tau$  with an account of the optical theorem can be written in the form [14]<sup>4</sup>:

$$\frac{dS}{d\tau} = \sum \int \delta^{(4)}(P_F - P_I) \times \\ \ln \left( \frac{Z_{if}}{Z_{fi}} \right) (Z_{if} - Z_{fi}) W_{if} \prod_{i,f} dP dV \quad (13)$$

Therefore for thermodynamical equilibrium  $dS/d\tau = 0$  a fulfillment of the functional Boltzmann equations is a necessary condition:

$$Z_{if} - Z_{fi} = 0. \quad (14)$$

<sup>4</sup>for  $T$  - invariant as well as for  $T$  - nonvariant interactions.

#### 1.4. The Local Thermodynamic Equilibrium Of The System Of Electrically Charged Particles

The next statements are true.

**Statement 1.** *For the statistical system consisting of  $N$  sorts of identical particles to be in the state of LTE in a gravitational field it is necessary and sufficient that*

1. *the invariant function of these particles' distribution  $f_a(x, p)$  has the locally equilibrium form:*

$$f_a^0(x, p) = [\exp(-\gamma_a + (\xi, p)) \pm 1]^{-1}, \quad (15)$$

where

$$\xi^i = \xi^i(x) \quad (\xi, \xi) > 0 \quad (16)$$

is a timelike vector,

2. *the reduced chemical potentials  $\gamma_a(x)$  satisfy the set of chemical equilibrium conditions*

$$\sum_{A=1}^N \nu_A^k \gamma_A = 0, \quad (17)$$

Locally equilibrium distribution functions (15) are the sole positively defined solutions of the Boltzmann functional equations (14). Introducing with a help of timelike vector  $\xi^i(x)$  the macroscopic kinematic velocity [15],  $v^i$

$$v^i \equiv \frac{\xi^i}{\xi}, \quad \xi \equiv \sqrt{(\xi, \xi)}; \Rightarrow (v, v) = 1, \quad (18)$$

as well as the local temperature of the statistical system,  $\theta$ :

$$\theta(x) = \frac{1}{\xi(x)}, \quad (19)$$

and also introducing by means of the reduced chemical potential the chemical potential in the standard normalization:

$$\mu_a(x) = \gamma_a \theta, \quad (20)$$

let us write in standard denotation the equilibrium distribution function and the conditions of the chemical equilibrium:

$$f_a^0(x, p) = \left[ \exp \left( \frac{-\mu_a + (v, P)}{\theta} \right) \pm 1 \right]^{-1}; \quad (21)$$

$$\sum_{A=1}^N \nu_A^k \mu_A = 0. \quad (22)$$

#### 1.5. The Global Thermodynamic Equilibrium Of The System Of Electrically Charged Particles

The locally equilibrium distribution functions (15), (21) turn the integral of motion (11) to the null equation. In case when distribution functions (15) or (21) are the exact solutions of the kinetic equations (10) the statistical system is in the strict global thermodynamical equilibrium (GTE). At conditions of GTE the strict laws of conservation of each sort particles are fulfilled and the system entropy is strictly constant  $S = \text{Const}$ . The substitution of (15) to the kinetic equations (10) reduces them to the form:

$$[H_a, \phi_a] = 0, \quad (23)$$

where

$$\phi_a(x, P) = \gamma_a(x) + (\xi, P_a) \quad (24)$$

is an argument of the equilibrium distribution function (15). Thus to ensure GTE there should exist a linear integral of motion having  $\xi^i$  is a timelike vector. Resolving (23) we obtain a set of necessary and sufficient conditions of GTE existence GTE [2], [5, 6], [9, 13]:

$$\mathcal{L}_\xi g_{ik} = \begin{cases} 0, & m_a \neq 0; \\ \varrho g_{ik}, & m_a = 0, \end{cases} \quad (25)$$

$$e_a \mathcal{L}_\xi A_i = \gamma_{a,i}, \quad (26)$$

where  $\mathcal{L}_\xi$  is a Lie derivative with respect to the direction  $\xi$ ,  $\rho(x)$  – is an arbitrary scalar function.

Let us notice a very notable for the cosmology strict fact that follows from (25) and have a power of theorem.

**Statement 2.** *If the rest masses of all the particles of the statistical system are equal to zero then the condition of the global thermodynamic equilibrium is possible if and only if the metrics is conformally stationary. If even a single sort of massive particles is present in the statistical system then the condition of the GTE of the statistical system is possible only for the stationary gravitational field.*

Thus if in the statistical system with electromagnetic interaction there is at least single sort of the particles with a non-zero rest mass then this particle's motion vector  $\xi^i$  should be a timelike Killing vector (25).

Next, for the cosmology it is of extremely great importance the property of the asymptotic conformal invariance of the relativistic theory of the statistical systems with electromagnetic interaction which was proved in the Author's work [10]:

**Statement 3.** *In the ultrarelativistic limit*

$$m_a/\langle p_a \rangle \rightarrow 0 \quad (27)$$

*the kinetic theory is asymptotically conformally invariant i.e. the kinetic equations with accuracy within values of the second order of smallness of parameter (27) are invariant with respect to the conformal transformations*

$$d\bar{s}^2 = \rho^2 ds^2; \quad (28)$$

$$\bar{A}_i = A_i + \partial_i \varphi; \quad (29)$$

$$\bar{P}_i = P_i - e \partial_i \varphi. \quad (30)$$

As a result of the asymptotic conformal invariance of the relativistic kinetic theory the state of GTE is asymptotically reachable in the ultrarelativistic cosmological plasma. The violation of this state in consequence of plasma cooling and violation of the condition (27) triggers the mechanism of spontaneous breaking of different symmetries in the Universe including the barion symmetry. Such mechanisms were investigated by several authors in 1970 and 1980 [16], [17] and others and more strict kinetic theory of the barion symmetry spontaneous breaking was developed in the Author's papers [18, 19, 20, 21].

### 1.6. The Equilibrium Self-Gravitating Statistical Systems

Since all moments of the equilibrium distribution function are determined via scalars  $\xi^2$ ,  $\lambda$ ,  $\Phi$  and tensors  $\xi^i$ ,  $g^{ik}$ ,  $\xi^i \xi^k$ , ..., then the conservation laws for the moments of the distribution function are also fulfilled [13]:

$$\mathbb{L}_\xi n_a^i = 0; \quad (31)$$

$$\mathbb{L}_\xi T^{ik} = 0 \quad (32)$$

etc.

In consequence of the first group of equations (25) the following components of Riemann-Ricci tensor and Einstein tensor are conserved along the direction  $\xi^i$ :

$$\mathbb{L}_\xi R_{ijkl} = 0; \quad \mathbb{L}_\xi R_{ij} = 0; \quad \mathbb{L}_\xi G_{ij} = 0. \quad (33)$$

Therefore in consequence of the Einstein equations there should be fulfilled the next relations:

$$\mathbb{L}_\xi T_{ij} = 0. \quad (34)$$

This allows us to carry out the strict classification of all the equilibrium self-gravitating configurations with electromagnetical interaction [22]:

**Statement 4.** *Let  $\xi^i(x)$  is a timelike Killing vector in the equilibrium distribution function (15) and the gravitational field of the equilibrium configuration allows group of motions of the  $r$ ,  $G^r$  order so that:*

$$\mathbb{L}_\lambda g_{ik} = 0; \quad \lambda = \overline{1, r}, \quad (35)$$

where  $\mathbb{L}_\lambda \equiv \mathbb{L}_{\xi_\lambda}$ . Let us expand the Killing vector by vectors of  $G^r$  group:

$$\xi^i = \sum_{\lambda=1}^r \alpha_\lambda \xi_\lambda^i; \quad (\alpha_\lambda = \text{Const}). \quad (36)$$

Then for the global equilibrium of the self-gravitating statistical systems of particles with electromagnetic interaction it is necessary and sufficient that the Killing motion vector of the system  $\xi^i$  and tensor of the electromagnetic field  $F_{ik}$  are conserved during motions along the group  $G^r$ :

$$\mathbb{L}_\lambda \xi^i = 0 \Rightarrow \nabla_k \sum_{\lambda, \lambda' \neq [\lambda, \lambda']} \xi_\lambda^k \xi_{\lambda'}^i; \quad (37)$$

$$\mathbb{L}_\lambda F_{ik} = 0; \quad \lambda, \lambda' = \overline{1, r}. \quad (38)$$

In the language of generators of Lie group  $X_\lambda = \xi_\lambda^i \partial_i$  the condition (37) means that the group of motion of the charged particles' equilibrium self-gravitating configuration should have a timelike center [23].

## 2. The Global Thermodynamic Equilibrium Of The Statistical Systems With Scalar Interaction

The common-relativistic kinetic theory of the statistical systems with scalar interaction was formulated by the Author and above cited papers [10, 11, 12, 13] and in more modern form taking into account phantom scalar fields this theory was reformulated in [24] – [28]. In particular in these papers it is shown that the invariant Hamilton function of the particle in a scalar field  $\Phi$  can be chosen in the next form accurate within automorphisms:

$$H(x, P) = \frac{1}{2} [(m^*)^{-1}(\Phi)(P, P) - m^*(\Phi)] = 0, \quad (39)$$

where

$$m^*(\Phi) = |m + q\Phi| \equiv |\phi|, \quad (40)$$

$m$  is a bare mass of a particle,  $q$  is particle scalar charge. Function  $m^*(\Phi)$  can be named as particle effective mass. Thus the normalization ration for the generalized momentum is fulfilled:

$$(P, P) = m^{*2}. \quad (41)$$

The kinetic equations (10) at that conserve their form. The relations for the entropy (12) and its derivative (13) also conserve their form thereby the formula for the locally equilibrium distribution function is also conserved (15). The substitution of this function in the kinetic equations (10) with an account of the explicit form of the Hamilton function (39) leads to the next relation:

$$-\gamma_{a,i} \frac{P^i}{m_a^*} + \frac{1}{m_a^*} \xi_{i,k} P^i P^k - \text{sgn}(\phi_a) q_a \xi^i \Phi_{,i} = 0,$$

for the fulfillment of which as a consequence of the generalized momentum the necessary and sufficient conditions are:

$$\partial_i \gamma_a = 0 \Rightarrow \gamma_a = \text{Const}; \quad (42)$$

$$\int_{\xi} g_{ik} = \sigma g_{ik}; \Rightarrow \quad (43)$$

$$\sigma = \int_{\xi} \ln m_a^*. \quad (44)$$

The substitution of (44) in (43) leads to the condition:

$$\int_{\xi} \frac{1}{m_a^*} g_{ik} = 0. \quad (45)$$

In the case if particle bare masses  $m \equiv m_a$  in (40) are different from zero there remains the single possibility of resolution of the relations (43) – (44) as a consequence of the difference of the bare masses and particle charges:

$$\int_{\xi} g_{ik} = 0; \quad (46)$$

$$\int_{\xi} \Phi = 0. \quad (47)$$

Exactly this conclusion was made even in former works of the Author like [13] and others. Thus also in the case of scalar interactions for the maintenance of the thermodynamic equation it was required the stationarity of the gravitational and scalar fields. However this conclusion is not absolutely strict one. In case if bare masses of all particles are equal to zero there exists one more possibility of the global thermodynamic equilibrium since in this case it is  $m_a^* = |q_a \Phi|$ :

$$\int_{\xi} \frac{1}{\Phi} g_{ik} = 0. \quad (48)$$

Apparently Author halted with investigation of this case in 1982-1983 due to extreme exoticism of the suggestion about all bare masses of elementary particles could be equal to zero. However in 2014 after the discovery of the Higgs boson such a suggestion is now natural. Thus let us formulate the statement about the global thermodynamic equilibrium of the statistical system with scalar interaction.

**Statement 5.** *If the effective masses of all particles of the statistical systems are generated exclusively by the scalar interaction then for the global thermodynamic equilibrium of such system it is necessary that the conformal metrics of the space-time*

$$\bar{g}_{ik} \equiv \frac{1}{|\Phi|} g_{ik} \quad (49)$$

*is a stationary one and chemical potentials of the statistical system are constant.*

This property of the statistical systems with a scalar interaction can bring the far-reaching consequences for the cosmology. Let us notice that if consider for instance the cosmological models with a constant curvature:

$$ds^2 = a^2(\eta)(d\eta^2 - dl^2), \quad (50)$$

where  $dl^2$  is a metrics of the 3d space of the constant curvature then the condition (49) brings us to the following relation for the potential of the scalar field:

$$|\Phi| \sim a^2(t). \quad (51)$$

In such system the strict global thermodynamic equilibrium will always be maintained. Let us notice that the presence of vector interactions can not change this conclusion and this one can find in the above cited Author's works. The presence of such a stage in the evolution of the Universe can lead to the restoration of the barion and other symmetries at this stage.

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